



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

138. Proposed by HARRY S. VANDIVER, Bala, Pa.

Show that the number of solutions in positive integers for x , y , and z of $x^3 + 2y^3 + 4z^3 - 6xyz = 1$ is infinite.

Solution by the PROPOSER.

Suppose, for the present, that there is one solution of the proposed equation. Let $x=a$, $y=b$, $z=c$, satisfy it. Then

$$a^3 + 2b^3 + 4c^3 - 6abc = 1. \quad \text{Whence } [a^3 + 2b^3 + 4c^3 - 6abc]^n = 1.$$

Then whatever values of x , y , and z we take consistent with $x^3 + 2y^3 + 4z^3 - 6xyz = 1$, we will have

$$[a^3 + 2b^3 + 4c^3 - 6abc]^n = x^3 + 2y^3 + 4z^3 - 6xyz,$$

which is satisfied by the following assumptions:

$$\begin{aligned} x + y\sqrt[3]{2} + z\sqrt[3]{4} &= [a + b\sqrt[3]{2} + c\sqrt[3]{4}]^n \\ x + \omega y\sqrt[3]{2} + \omega^2 z\sqrt[3]{4} &= [a + \omega b\sqrt[3]{2} + \omega^2 c\sqrt[3]{4}]^n \quad (1) \\ x + \omega^2 y\sqrt[3]{2} + \omega z\sqrt[3]{4} &= [a + \omega^2 b\sqrt[3]{2} + \omega c\sqrt[3]{4}]^n \end{aligned}$$

(where $\omega^3 = 1$) as may be seen by multiplying together the equations, term for term. (1) gives on expanding

$$x + y\sqrt[3]{2} + z\sqrt[3]{4} = A + B\sqrt[3]{2} + C\sqrt[3]{4}$$

where A , B , and C are positive integers.

Hence, we must have $x=A$, $y=B$, $z=C$.

(2) and (3) give likewise

$$\begin{aligned} x + \omega y\sqrt[3]{2} + \omega^2 z\sqrt[3]{4} &= A + \omega B\sqrt[3]{2} + \omega^2 C\sqrt[3]{4} \\ x + \omega^2 y\sqrt[3]{2} + \omega z\sqrt[3]{4} &= A + \omega^2 B\sqrt[3]{2} + \omega C\sqrt[3]{4} \end{aligned}$$

In each of these we will have as in (1) $x=A$, $y=B$, $z=C$.

Therefore, x , y , and z have an infinite number of integral values depending on the value of n ; provided, as we assumed at the start, that they have one set of values.

This one set of values is $x=1$, $y=1$, $z=1$; for $1^3 + 2 \times 1^3 + 4 \times 1^3 - 6 = 1$.

Let us find, for instance, the values of x , y , and z for $n=2$, having given that $a=1$, $b=1$, $c=1$, we have

$$x + y\sqrt[3]{2} + z\sqrt[3]{4} = [1 + \sqrt[3]{2} + \sqrt[3]{4}]^2, \text{ or } x=5, y=4, z=3,$$

which satisfy. Putting $n=3$, we find another set, and so on.

The method used in the solution of this problem is a particular case of a general principle applicable to the solution of many Diophantine questions.